AN EVALUATION OF CERTAIN FORMS OF MOMENTUM TRANSFER IN THE 26-MONTH OSCILLATION

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ABSTRACT

The fundamental equations are linearized by the method of small perturbations for the conditions of hydrostatic equilibrium and geostrophic balance of meridional forces. In a first case, Fickian momentum diffusion is used; in a second case, a shear- and latitude-dependent eddy viscosity is used. A wave solution for the zonal wind oscillation, having in meridional profile the shape of a probability curve, is substituted in the equation of zonal motion. This procedure leads first to conditions on parameters, next to a solution for the meridional wind, and, through the remaining perturbation equations, finally to solutions for the remaining perturbation quantities. The temperature perturbation follows from the thermal wind equation and has the same probability curve shape in meridional profile as does the zonal wind disturbance.

The conditions on parameters and the resulting solutions show that the assumed forms of eddy diffusion of momentum cannot under any circumstance account for momentum changes in the non-attenuating layer (above about 25 km.), but may, to the extent that mean vertical motion over the equator is negligible, be applied at the levels where the oscillation attenuates downward (below about 25 km.). In this case air drifts equatorward during westerlies and poleward during easterlies, and the oscillation consists of downward-propagating cells in meridional cross-section.

1. INTRODUCTION

The problem of accounting for westerly momentum over the equator is fundamental to an understanding of the 26-month oscillation. Despite an ever-increasing description of the fundamental fields of oscillation, the mode of equatorward transfer of westerly momentum remains unspecified, except, residually, as lying somewhere in the mysteries of the Reynolds stresses (Reed [5]). The apparent small scale of the eddies accounting for the transfer of momentum, and the sparsity of observations in the Tropics, have thus far prevented the detection of correlation between zonal and meridional wind fluctuations.

Reed [6] has now extended the description to include meridional and vertical fields of motion. In brief, the oscillation is now seen to consist of downward-propagating cells of circulation in meridional cross-section, with air drifting equatorward during westerlies and poleward during easterlies. Vertical motion over the equator is positive between the level of westerlies and the easterlies above them, and negative between the level of westerlies and the easterlies below.

This more complete description now assists in making a diagnostic evaluation of specific forms for the Reynolds stress terms in the equation of zonal motion.

2. PURPOSE

The purpose of this paper is to determine to what extent two common forms of momentum diffusion can describe the observed changes of zonal momentum in the 26-month oscillation. The first form of momentum diffusion analyzed is Fickian; in the second case a shear- and latitude-dependent eddy viscosity is analyzed. The method of small perturbations is used, although, where relevant, the usually neglected nonlinear terms are assessed. Two layers are considered: a layer above about 25 km. where the amplitude is nearly invariant with height, and a layer below about 25 km. where the amplitude attenuates downward.

The procedure to be followed here is essentially diagnostic. A function for the best known oscillation, that of zonal wind, will be substituted in the equation of motion. This leads to an evaluation of the assumed form of the momentum diffusion, and, if the correct form of zonal wind is used, it is possible to deduce from the complete set of equations the oscillations of temperature, vertical and meridional motion, and the heating function.

3. THEORY

The undisturbed equatorial stratosphere will be assumed to be in hydrostatic equilibrium. Hence

$$\frac{\partial P}{\partial z}+gQ=0, \tag{1}$$

where P and Q are undisturbed pressure and density, respectively. The undisturbed motion includes a zonal component as shown by Reed and Rogers [7]. Vertical and meridional mean motions must also exist, but their magnitudes have not yet been determined. Hence the mean motion is specified, for the present,

$$\mathbf{V}=\mathbf{i}U+\mathbf{j}V+\mathbf{k}W, \tag{2}$$

where U, V, and W all may depend on y and z. Undisturbed temperature certainly depends on height. If U depends on height and the undisturbed motion is

[1] The research reported in this article has been supported by the Office of Naval Research, under Contract Nonr 2173(02) at the University of Arizona, and by a grant from the National Science Foundation at the Department of Atmospheric Sciences of the University of Washington.

geostrophic, then undisturbed temperature must also depend on latitude. Perturbation quantities are related to instantaneous and undisturbed quantities according to

$$u^*=U+u \text{ (zonal wind)} \qquad p^*=P+p \text{ (pressure)}$$
$$v^*=V+v \text{ (meridional wind)} \qquad \rho^*=Q+\rho \text{ (density)}$$
$$w^*=W+w \text{ (vertical wind)} \qquad T^*=T+\tau \text{ (temperature)}. \quad (3)$$

Here asterisks denote instantaneous values, capitals without asterisks denote undisturbed values, and lower case letters denote perturbation quantities. A perturbation equation of zonal motion averaged around the earth in the vicinity of the equator may be obtained by the usual procedures of the perturbation method. Here we assume vertical momentum flux proportional to vertical gradient of momentum. This is consistent with the nearly constant downward propagation of phase, and no compelling evidence can be found for a different form.

The form of meridional diffusion is not so clear, and a major purpose of this paper is to examine the consequences of two substantially different assumptions concerning the form of the meridional diffusion. Because the oscillation is propagating downward and attenuating downward in the lower tropical stratosphere, and since Fickian diffusion is known to dissipate extremes, it seems conceivable that Fickian diffusion might account for momentum changes in the lower stratosphere. This will be treated as a first case. In a second case, a shear- and latitude-dependent meridional eddy viscosity will be assumed.

If meridional and vertical eddy fluxes of momentum may be expressed in terms of shears and eddy viscosity coefficients, the perturbation equation for the zonal motion may be written as

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}+W\frac{\partial u}{\partial z}+\frac{\partial U}{\partial y}v+\frac{\partial U}{\partial z}w-\frac{1}{Q}\frac{\partial}{\partial y}\left(QK_{my}\frac{\partial u}{\partial y}\right)-\frac{1}{Q}\frac{\partial}{\partial z}\left(QK_{mz}\frac{\partial u}{\partial z}\right)-fv=0, \quad (4)$$

where K_{my} and K_{mz} are eddy viscosity coefficients for northward and upward diffusion, respectively, and f is the Coriolis parameter. The Coriolis term in w has been neglected in comparison with the remaining terms. The term, $-w(\partial U/\partial z)$, may be compared with $\partial u/\partial t$ by setting $w\approx 0.01$ cm. sec.$^{-1}$ (Reed [6]), $\partial U/\partial z\approx -0.6$ m. sec.$^{-1}$ km.$^{-1}$ (Reed and Rogers [7]), and $\partial u/\partial t\approx 0.4\times 10^{-5}$ m. sec.$^{-2}$. Hence the fifth term is two orders of magnitude smaller than the rate of change of westerly momentum per unit mass, which is known to be important. The meridional variation of undisturbed density may also be neglected. No direct information regarding the magnitudes and variation of K_{my} and K_{mz} are available, unless these are identified with particulate diffusion coefficients. However, the occurrence of westerlies over the equator indicates the danger of equating momentum coefficients to particulate coefficients. Here we assume for simplicity that QK_{mz} is constant with height. The effect of height variation of this quantity may readily be inferred. To achieve a completely Fickian diffusion, we assume that K_{my} is constant. Hence (4) becomes

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}+W\frac{\partial u}{\partial z}+\frac{\partial U}{\partial y}v-K_{my}\frac{\partial^2 u}{\partial y^2}-K_{mz}\frac{\partial^2 u}{\partial z^2}-fv=0. \quad (5)$$

The long period of the oscillation strongly suggests geostrophic balance of the meridional pressure gradient force, and Reed [4] has verified from observations and the thermal wind equation that the motion is in fact geostrophic. Hence the northward component of the equation of motion may be written as

$$fu+\frac{1}{Q}\frac{\partial p}{\partial y}=0. \quad (6)$$

The long period and large scale suggest also that the oscillation is in hydrostatic equilibrium. This assumption has been used with success in the study of baroclinic instability. Hence the perturbation equation of vertical motion becomes

$$\frac{\partial p}{\partial z}+g\rho=0 \quad (7)$$

in view of (1). Equation (5) does not, of course, exclude vertical motion, which must appear in non-negligible terms of the equations of continuity and thermodynamic energy. The perturbation equation of state may be written as

$$\frac{p}{P}-\frac{\rho}{Q}-\frac{\tau}{T}=0. \quad (8)$$

Now equations (6), (7), and (8) may be combined to yield a thermal wind equation which replaces all three:

$$\frac{\partial u}{\partial z}+\frac{g}{fT}\frac{\partial \tau}{\partial y}=0. \quad (9)$$

Small terms, including one in lapse rate, have been neglected.

For completeness and later reference we may also write down the perturbation equations of continuity and thermodynamic energy. The perturbation equation of continuity averaged over longitude may be written as

$$\frac{\partial w}{\partial z}+\frac{1}{Q}\frac{\partial Q}{\partial z}w+\frac{\partial v}{\partial y}=0. \quad (10)$$

In obtaining a thermodynamic energy equation, we equate individual change of potential temperature to the divergence of turbulent enthalpy flux plus a heating function which includes contributions from radiation and molecular heat conduction, the latter probably negligible. The individual derivative of potential temperature is expanded, and, in the local and horizontal advection terms, the potential temperature is approximated by ambient temperature. In the term for vertical diffusion, the quantity $\rho\theta T^{-1}K_{hz}$, where K_{hz} is the vertical eddy conductivity, is assumed constant for simplicity and similarity of eddy viscosity and eddy conductivity terms. In the term for horizontal diffusion, potential temperature is replaced by ambient temperature, and K_{hy}, the northward eddy conductivity, is assumed constant. Thus the perturbation thermodynamic energy equation becomes

$$\frac{\partial\tau}{\partial t}+V\frac{\partial\tau}{\partial y}+W\frac{\partial\tau}{\partial z}+v\frac{\partial T}{\partial y}+(\Gamma-\gamma)w-K_{hy}\frac{\partial^2\tau}{\partial y^2}-K_{hz}\frac{\partial^2\tau}{\partial z^2}-h=0, \tag{11}$$

where Γ and γ are adiabatic and ambient lapse rates, respectively, and h is the heating function.

The system of perturbation equations now consists of (5), (9), (10), and (11). If h is specified from radiation theory, the dependent variables are u, v, w, and τ. In principle, three of these could be eliminated and a differential equation derived for the amplitude of the fourth. These equations turn out to be cubic or quartic with variable coefficients, a situation which, in the absence of a clear understanding of the form of momentum diffusion, or of the proper boundary conditions, makes analysis extremely difficult as well as inconclusive. It seems more profitable at this stage to specify the variable which is best known from observations, substitute and determine those remaining variables such as v and w which are not easily measured directly. In this way the complete perturbation structure may in principle be determined. The best known variable is, of course, the zonal wind. If it is substituted in the zonal momentum equation, (4) or (5), then a solution for meridional wind is obtained. To the extent that the correct terms and forms of terms have been included, the solution for v will agree with conclusions drawn by Reed [6] from observations. These indicate equatorward drift during westerlies and poleward drift during easterlies, with magnitudes of a few centimeters per second. Substitution of the observed zonal wind in (4) or (5) also leads to conditions on various parameters of the problem. The sense of these conditions also serves to evaluate the assumed form of the terms in the momentum equation. Since momentum changes in the oscillation are the principal problem, the purpose of this paper will be to follow the aforementioned procedure and strive to determine what forms for momentum diffusion lead to physically realistic conditions on parameters and solutions for the meridional wind.

The zonal wind oscillation is most intense and most accurately reported within 20° of the equator. The simplest zonal wind oscillation which describes the observations with good accuracy is a downward-propagating and downward-attenuating disturbance whose latitudinal profile has the form of the normal probability curve,

$$u=u_{EM}e^{-ky^2}e^{\alpha_a z+i(\alpha_p z+\nu t)} \tag{12}$$

where u_{EM} is the magnitude of u over the equator at a reference level where z is zero, k is a real, positive constant which determines the rate of decrease of amplitude with increasing distance from the equator, α_a is a constant which determines the rate of downward attenuation, α_p is the positive wave number, and $\nu=2\pi$ (26 mo.)$^{-1}$ is the frequency. Equation (12) specifies a simultaneous phase at all latitudes. Reed's and Rogers' [7] analysis of observations shows phase varying by as much as 3 mo. within 20° of the equator. This variation is small compared with the period (26 mo.) and will be neglected here for simplicity. However, a complex k is readily handled mathematically. The choice of the probability curve for the meridional profile is justified not only by direct comparison with amplitudes presented by Reed and Rogers [7], but also by comparing with observations the temperature disturbance which it implies through the thermal wind equation. Thus, from (9), by partial (meridional) integration,

$$\int_0^\tau d\tau=-\frac{T}{g}\int_{y_0}^{y} f\frac{\partial u}{\partial z}\partial y=-\frac{T(\alpha_a+i\alpha_p)u_E}{g}\int_{y_0}^{y} fe^{-ky^2}\partial y, \tag{13}$$

where u_E is u over the equator, and where the temperature disturbance has been assumed to vanish at latitude y_0. Within about 20° of the equator, $f\approx 2\Omega a^{-1}y$, where Ω is the earth's angular velocity and a is earth radius. Hence

$$\tau=-\frac{2\Omega T(\alpha_a+i\alpha_p)u_E}{ga}\int_{y_0}^{y}\xi e^{-k\xi^2}d\xi, \tag{14}$$

where ξ is a dummy variable corresponding to y. Integration yields

$$\tau=-\frac{\Omega T}{gak}(\alpha_a^2+\alpha_p^2)^{1/2}e^{i\omega}u_E(e^{-ky_0^2}-e^{-ky^2}), \tag{15}$$

where

$$\omega=\arctan \alpha_p/\alpha_a. \tag{16}$$

Hence the temperature disturbance precedes the zonal wind disturbance by $(13\omega/\pi)$ months, has in meridional profile the shape of a normal probability curve, is positive for $y<y_0$ and negative for $y>y_0$. All these features are consistent with a model presented by Reed [6] on the basis of observations. Of course, the temperature profile is not so reliably known as the zonal wind profile, but it is readily verified that an exponential, rather than a probability, profile for the zonal wind disturbance, leads, through the thermal wind equation, to a temperature profile which is difficult to adjust quantitatively or qualitatively to observations. In (15) if we set $\alpha_a=\alpha_p=2\pi/26$ km., $T=210°$ K., $k=5\times10^{-3}$ (lat. deg.)$^{-2}$, $u=20$ m. sec.$^{-1}$ at the equator, $y_0=15$ lat. deg. (Reed [6]), the temperature amplitude over the equator becomes 2.7° C., a value in good agreement with Reed's [4] results.

Therefore, (12) is a faithful representation of the observed zonal wind disturbance, and when it is substituted in (5), defects which arise in the conditions on constants or in the solution for v must then be traced to the form of the momentum diffusion terms.

Substitution of (12) in (5) yields

$$[W\alpha_a+2kK_{my}(1-2ky^2)-K_{mz}(\alpha_a^2-\alpha_p^2)-2kV +i(\nu-2\alpha_a\alpha_pK_{mz}+W\alpha_p)]u-\left(f-\frac{\partial U}{\partial y}\right)v=0. \tag{17}$$

It is now necessary to specify $\partial U/\partial y$. Reed and Rogers [7] show the amplitude of mean zonal wind as a function of latitude. Between 25 and 50 mb. the mean zonal wind is a few meters per second easterly over the equator, of the order of 10 to 15 m. sec.$^{-1}$ easterly around 10° N., and less easterly farther north. Examination of magnitudes shows that $\partial U/\partial y$ may not be neglected in comparison with f within 10° of the equator, but that $f \gg \partial U/\partial y$ for latitudes greater than 10°. The zonal wind profiles as sketched by Reed and Rogers are rounded at the equator, and, with sufficient accuracy for our purposes, may be specified in the form

$$U=U_\infty+U_1e^{-ly^2}, \tag{18}$$

where $U_\infty(<0)$, $U_1(>0, <-U_\infty)$, and $l(>0)$ are constants. Hence

$$\frac{\partial U}{\partial y}=-2lU_1ye^{-ly^2}. \tag{19}$$

Calculations by Murgatroyd and Singleton [3] show mean meridional motions are poleward and symmetric about the equator in the region from the tropopause to 35 km. It will therefore be assumed that V vanishes at the equator. Now, by taking the limit as $y \rightarrow 0$ in (17), the term in v vanishes, and we obtain the following conditions on parameters:

$$K_{mz}(\alpha_a^2-\alpha_p^2)-2kK_{my}-W_E\alpha_a=0, \tag{20}$$

$$\nu-2\alpha_a\alpha_pK_{mz}+W_E\alpha_p=0, \tag{21}$$

where the subscript E denotes evaluation at the equator. These expressions show that the various parameters cannot be chosen arbitrarily. Of the parameters in (20) and (21), the best known are ν, α_p, α_a, and k. The sense of W_E is almost certainly positive in view of the deficiency of total ozone in the Tropics and the low temperatures in the lower tropical stratosphere. From the adiabatic cooling required to offset diabatic heating, Murgatroyd and Singleton have estimated W_E to be of the order of 0.05 to 0.1 cm. sec.$^{-1}$ in the region from 18 to 35 km.

In the lower stratosphere, where the oscillation is attenuating downward, α_a is of the same order as α_p. For $\alpha_a=\alpha_p[=\nu/c$, where $c\approx 1$ km. mo.$^{-1}$ is the wave speed and $\nu=2\pi$ (26 mo.)$^{-1}]$, we find from (21) that $K_{mz}=8\times10^3$ cm.^{2} sec.$^{-1}$ and 1.8×10^4 cm.^{2} sec.$^{-1}$ for $W_E=0$ and 0.05 cm. sec.$^{-1}$, respectively. The increase of K_{mz} with increasing W_E traces physically to the necessity of a larger diffusivity to conduct the momentum downward at a fixed rate against the upward motion of the medium. These values for K_{mz} are only slightly larger than the particulate diffusion coefficients of the order of 10^3 to 10^4 cm.^{2} sec.$^{-1}$ deduced from radioactivity diffusion by Friend, et al. [2]. There is thus no particular reason to reject a Fickian form for vertical momentum diffusion in the attenuating layer. In the non-attenuating layer in the middle stratosphere, $\alpha_a=0$, and in this case $W_E=-\nu\alpha_p^{-1}=-c$. This suggests that diffusion is here unimportant but gives no indication of the correctness of the formulation.

The condition (20) involves meridional diffusion and is, unlike (21), difficult to reconcile with known facts. If K_{my} and W_E are both positive, then (14) requires $\alpha_a>\alpha_p$. Thus (14) cannot apply to the non-attenuating layer. Analysis by Reed [6] indicates that α_a and α_p are of the same order in the attenuating layer. In order to be consistent with both (14) and the condition that α_a be not too different from α_p, it is necessary in (14) that K_{my} be about 10^8 cm.^{2} sec.$^{-1}$ for $W_E=0$, or somewhat smaller than the values determined from the speed of radioactivity transport for the meridional particulate diffusion coefficient. The latter values are of the order of 10^9 cm.^{2} sec.$^{-1}$ If W_E is allowed to increase from zero, it becomes increasingly difficult to obtain an intuitively reasonable value for K_{my}. For $W_E=c=1.15$ km. mo.$^{-1}=0.045$ cm. sec.$^{-1}$ and $\alpha_a=2\alpha_p$ (approximately the maximum α_a allowable in the attenuating layer), it follows that K_{my} is negative. Thus, if the sense of undisturbed vertical motion in the tropical stratosphere is upward, the results suggest that meridional diffusion of momentum cannot be Fickian.

Despite the restriction on parameters, a further exploration of the present case is instructive and necessary since, even if it is in fact positive, the correct value for W_E is still uncertain, and a negative K_{my} may only reflect the existence of eddies which transfer momentum up the gradient. It will be shown, however, that $K_{my}<0$ leads to meridional winds which are of incorrect sign, while positive values less than 10^9 cm.^{2} sec.$^{-1}$ lead to the correct sign but somewhat smaller magnitudes than indicated by observations. Substitution of the conditions (20) and (21) in (17) yields

$$\left(f-\frac{\partial U}{\partial y}\right)v=[\alpha_a(W-W_E)-4k^2K_{my}y^2 +2kyV+i\alpha_p(W-W_E)]u. \tag{22}$$

It is seen that v vanishes at the equator.

For negligible mean motions (22) becomes

$$v=-\frac{4k^2K_{my}y^2u}{f-\frac{\partial U}{\partial y}}, \tag{23}$$

where we assume K_{my} is a positive quantity. Inspection of (23) with reference to (12) and (19) shows that meridional velocity vanishes at the equator. At other latitudes, equatorward drift is associated with westerlies and poleward drift with easterlies. These are properties of the model of observed structure presented by Reed [6]. Within about 20° of the equator, $f\approx 2\Omega a^{-1}y$, and, in view of (19), it is accurate to write

$$v=-\frac{2k^2K_{my}yu}{\Omega a^{-1}+lU_1e^{-ly^2}}. \tag{24}$$

Poleward of roughly 7° lat., the second term in the denominator is negligible, and v reaches a maximum magnitude where the product yu reaches a maximum. Partial differentiation with respect to y and reference to (12) show that this maximum occurs at a y_m which satisfies

$$y_m=\frac{1}{\sqrt{2k}}. \tag{25}$$

For $k\approx 5\times 10^{-3}$ (lat. deg.)$^{-2}$, it follows that $y_m\approx 10°$. For $K_{my}=10^9$ cm.^{2} sec.$^{-1}$ and $u_{EM}=25$ m. sec.$^{-1}$, maximum meridional winds are of the order of 5 cm. sec.$^{-1}$, which agrees in magnitude with Reed's [6] values. One may now consider substituting for v in the equation of continuity, (10), and then integrating to obtain w, then, finally, substituting for τ, v, and w in (11) to derive a heating function.

Unfortunately, $K_{my}=10^9$ cm.^{2} sec.$^{-1}$ requires, because of (20), α_a to be substantially larger than α_p. Reed [6] indicates that α_a may at most be twice α_p, and this only in the lowest part of the attenuating layer. This is sufficient for (20) to yield 10^9 cm.^{2} sec.$^{-1}$ for K_{my} only if W_E vanishes or is negative. Since we have already noted that W_E is probably positive, it is very questionable if K_{my} may be chosen as a positive value sufficiently large to yield meridional velocities of the order of a few centimeters per second.

If mean motions exist, then not only does the condition (20) possibly yield a negative K_{my}, but also the functional relationship (22) between u and v becomes more complex. All terms in (22) may be comparable. According to Murgatroyd and Singleton [3], W is probably maximum positive over the equator, while V is probably zero at the equator and increases poleward within the Tropics. The phase difference between u and v will depart from 180° (13 mo.) with increasing distance from the equator.

We may conclude that when the probable mean motions are introduced, it becomes impossible to obtain a reasonable condition on the parameters or a reasonable solution for meridional velocity under the assumption of Fickian diffusion of momentum. Physically the difficulty in the attenuating layer lies in the circumstance that upward motion gives negative advection, so that K_{my} must be negative in order to concentrate westerly momentum. While a negative K_{my} is not of itself alarming, the subsequent solution for v does not appear realistic.

The possibility must also be considered that the nonlinear terms neglected in the perturbation method may in fact not be of higher order and negligible. The terms are $v(\partial u/\partial y)$ and $w(\partial u/\partial z)$. The first vanishes at the equator and cannot affect the results. The second term should be completely negligible because w is estimated by Reed [6] as roughly 0.01 cm. sec.$^{-1}$, or approximately one-tenth W.

The consequences of several forms of spatial variation of the meridional diffusion coefficient have been investigated, but none of these has led to more realistic conditions on parameters or a more realistic solution for the meridional wind disturbance.

Next, we will consider the consequences of assuming a *shear*-dependent eddy viscosity. Murgatroyd and Singleton [3] indicate almost no mean meridional motion across the equator in the lower and middle stratosphere. Reed [6] indicates that the meridional wind field in the 26-mo. oscillation also vanishes at the equator. These circumstances, together with vanishing of Coriolis and pressure gradient forces suggest that meridional eddy mixing may also be much smaller near the equator. Here it will be assumed that this is the case, that meridional eddy motions on all scales become very small as the equator is approached. Alternatively, it will be assumed that those eddies which effect a momentum exchange have increasingly restricted meridional displacements as the equator is approached. In complete analogy with the case of vertical eddy transfer of momentum near the ground for adiabatic conditions, we assume that mixing length is proportional to distance from the equator and that meridional velocity fluctuations are proportional to the mixing length and the meridional shear. Thus

$$K_{my}=\pm\kappa y^2\frac{\partial u}{\partial y}, \tag{26}$$

where the positive or negative sign applies when $\partial u/\partial y$ is positive or negative, respectively. In the adiabatic case near the earth's surface, y would be replaced by z and the constant κ would be 0.16, the square of the von Karman constant. Whether it may be so identified in the present case is doubtful, but it will be assumed to have the same order of magnitude.

If the meridional momentum diffusion term in the equation of motion is written out for the condition (26) and linearized by the method of small perturbations, and if the assumptions regarding all other terms in (4) are repeated, the linearized equation of motion becomes

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}+W\frac{\partial u}{\partial z}+2\kappa\frac{\partial}{\partial y}\left(y^2\frac{\partial U}{\partial y}\frac{\partial u}{\partial y}\right)-K_{mz}\frac{\partial^2 u}{\partial z^2}-\left(f-\frac{\partial U}{\partial y}\right)v=0. \tag{27}$$

Substitution of (12) in (27) yields

$$\left[W\alpha_a-K_{mz}(\alpha_a^2-\alpha_p^2)-2kV-2\kappa\frac{\partial}{\partial y}\left(2ky^3\frac{\partial U}{\partial y}\right)+8\kappa k^2y^3\frac{\partial U}{\partial y}+i(\nu-2\alpha_a\alpha_pK_{mz}+W\alpha_p)\right]u=\left(f-\frac{\partial U}{\partial y}\right)v. \tag{28}$$

Now, if the limit is taken as $y\to 0$ in (28), and if the same assumptions as before are made with respect to the mean motion, the term in v vanishes, and we obtain the following conditions on parameters:

$$(\alpha_p^2-\alpha_a^2)K_{mz}+\alpha_aW_E=0, \tag{29}$$

$$2\alpha_a\alpha_pK_{mz}-\alpha_pW_E=\nu. \tag{30}$$

In these expressions K_{mz} and W_E are least well-known. Solving for them we obtain

$$K_{mz}=\frac{\nu\alpha_a}{\alpha_p(\alpha_a^2+\alpha_p^2)}, \tag{31}$$

$$W_E=\frac{\nu(\alpha_a^2-\alpha_p^2)}{\alpha_p(\alpha_a^2+\alpha_p^2)}. \tag{32}$$

As in the previous case, W_E is proportional to the excess of α_a over α_p. Since in the attenuating layer α_a is observed to be of the same order as α_p, it is implied that W_E is small or that the formulation of vertical momentum transfer is incorrect. An accurate determination of mean vertical motion is necessary in order to resolve the matter. It may also be noted that when $W_E=0$ identical expressions for K_{mz} are obtained from (31) and (21).

Substitution of the conditions (31) and (32) in (28) yields

$$\left(f-\frac{\partial U}{\partial y}\right)v=\left[\alpha_a(W-W_E)-2kV-2\kappa\frac{\partial}{\partial y}\left(2ky^3\frac{\partial U}{\partial y}\right)\right.$$
$$\left.+8\kappa k^2y^3\frac{\partial U}{\partial y}+i\alpha_p(W-W_E)\right]u. \quad (33)$$

Here, again, v vanishes at the equator. Away from the equator, the sign and phase of v depend on the relative magnitudes of the various terms.

For negligible mean motions (33) becomes, in view of (19) and the approximate form for f,

$$v=-\frac{8\kappa kU_1ly^2[2-(l+k)y^2]e^{-ly^2}u}{\Omega a^{-1}+lU_1e^{-ly^2}} \quad (34)$$

Extremely close to the equator, (34) reduces to

$$v=-\frac{16\kappa kU_1ly^2u}{\Omega a^{-1}+lU_1}. \quad (35)$$

If the meridional diffusion term in (27) is identified as the meridional derivative of the product of the eddy viscosity coefficient and latitudinal shear, then the coefficient of eddy viscosity may be identified as $\pm 2\kappa y^2(\partial U/\partial y)$. Substitution of (19) and numerical values, including $\kappa=0.16$, shows that K_{my} exceeds 10^9 cm.^{2} sec.$^{-1}$ poleward of about 2° latitude. Hence (34) and (35) should not be applied poleward from that latitude, beyond which K_{my} no longer is related to distance from the equator.

Examination of (34) and (35) shows that v vanishes at the equator. Elsewhere air drifts equatorward during westerlies and poleward during easterlies. This result agrees with the previous development (when mean vertical motion is neglected) and with Reed's [6] measurements. However, v in the present case increases very rapidly with latitude. It follows that w would be proportionately larger than in the previous development.

If the mean vertical motion in the lower stratosphere is positive, having the magnitude computed by Murgatroyd and Singleton [3], then α_a becomes much larger than α_p, whereas $\alpha_a=2\alpha_p$ in the lower stratosphere appears to be a maximum excess of α_a over α_p. Moreover, the terms in (28) involving mean meridional circulation become important and the relationship of v to u becomes more complex and difficult to reconcile with Reed's [6] analysis of observations.

It is obvious that a careful determination of mean vertical motion over the equator is necessary to any theory of the 26-mo. oscillation. The cooling rates in the lower stratosphere presented by Murgatroyd and Singleton [3] are as much as four times as large as values presented by Davis [1]. Both of these computations involve averages over layers or smoothing in analysis. On the other hand, detailed radiative cooling profiles through the tropopause, obtained either from the radiative transfer equation or by means of the radiometersonde, show that radiative warming may occur in the vicinity of the tropopause, particularly when the tropopause is a level of minimum temperature as it is in the Tropics. (Physically, the warming can be traced to the Second Law of Thermodynamics; heat is transferred to the tropopause from its warmer surroundings.) We may therefore expect warming or else very small cooling at the tropical tropopause, with a diminished cooling extending some distance above the tropopause. For this condition, the mean vertical motion in the attenuating layer required to balance the radiative cooling is likely to be much smaller than indicated by Murgatroyd and Singleton, and the solutions for $W_E=0$ in the preceding development may be realistic.

4. CONCLUSION

Neither Fickian diffusion nor a shear- and latitude-dependent eddy viscosity appears applicable to the non-attenuating layer. In the attenuating layer these forms of momentum transfer may account for the observed structure if the mean vertical velocity over the equator is small or negligible. This latter condition may apply to the lowest part of the attenuating layer. The effect of mean vertical motion is crucial because of the associated advection; the applicability of various forms of eddy viscosity is contingent, in the method of analysis used here, on an accurate mean vertical velocity over the equator. This quantity is crucial to many studies and it is to be hoped that more accurate values will become available.

REFERENCES

1. P. A. Davis, "An Analysis of the Atmospheric Heat Budget," *Journal of the Atmospheric Sciences*, vol. 20, No. 1, Jan. 1963, pp. 5–22.
2. J. P. Friend, J. W. Feely, P. W. Krey, J. Spar, and A. Walton, "Discussion of High Altitude Sampling Program Results," Washington, D.C., Defense Atomic Support Agency (DASA 1300, vol. 3), 1961, 307 pp. (The Final Report on Contract DA-29-044-XZ-609; copies available from Isotopes Incorporated, 123 Woodland Ave., Westwood, N.J.)
3. R. J. Murgatroyd and F. Singleton, "Possible Meridional Circulations in the Stratosphere and Mesosphere," *Quarterly Journal of the Royal Meteorological Society*, vol. 87, No. 372, Apr. 1961, pp. 125–135.
4. R. J. Reed, "Evidence of Geostrophic Motion in the Equatorial Stratosphere," *Quarterly Journal of the Royal Meteorological Society*, vol. 88, No. 377, July 1962, pp. 324–327.
5. R. J. Reed, "On the Cause of the 26-Month Periodicity in the Equatorial Stratospheric Winds," *Meteorologisch Abhandlungen*, Band XXXVI, 1963, pp. 245–257.
6. R. J. Reed, "A Tentative Model of the 26-Month Oscillation," *Quarterly Journal of the Royal Meteorological Society*, vol. 90, No. 386, Oct. 1964, pp. 441–466.
7. R. J. Reed and D. G. Rogers, "The Circulation of the Tropical Stratosphere in the Years 1954–1960," *Journal of the Atmospheric Sciences*, vol. 19, No. 2, Mar. 1962, pp. 127–135.

[*Received September 10, 1964; revised October 30, 1964*]